Chapter 1: Introduction

Symbols introduced in this chapter

~ Similar

Δ Triangle

≅ Congruent

⊥ Perpendicular

° Degree

∠ Angle

m∠ Measure of angle

|| Parallel

<u>Points, lines, segments and rays</u>

Point: A point indicates position. It has no length, width or depth.

Line: A line is a straight, continuous set points that extends infinitely in either direction. Light travels in a straight line, unless passing from one medium into another. If two points *A* and *B* lie on a line, then we call the line *AB*. A line has no end points and in diagrams this is indicated by arrows, as in the following picture:

Planes: Planes are two-dimensional. A plane has length and width, but no height, and extends infinitely on all sides. Planes are thought of as flat surfaces, like a sheet of paper. A plane is made up of an infinite number of lines.

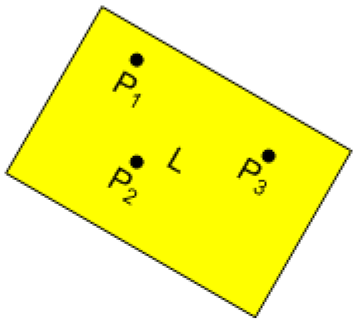
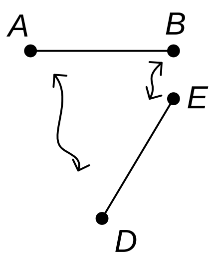


Fig 1.1: Plane *L* containing points P_1, P_2 and P_3.

Line segment: A line segment is a part of a line consisting of two points, called the end points, and the part of the line that is between them.

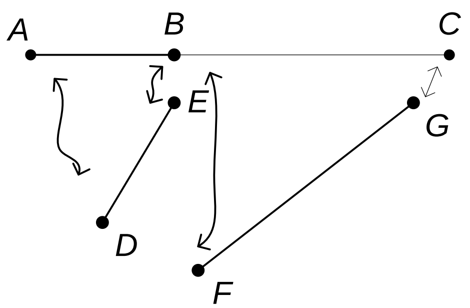
Congruence of line segments: Two segments are said to be congruent if, when we place one on top of the other, with one endpoint matching, the other endpoint also matches.[1]



Comparing line segments: Given segment *AB* and a point *P* in segment *AB* different from A and from B, we say: *segment AP is less than segment AB.* If segment CD is congruent to segment AP, we say: *segment CD is less than segment AB.*

Adding line segments: Suppose points A, B and C lie on a line in the order given (so B is between A and C). Then we say that segment AC is the sum of segments AB and BC. If segment DE is congruent to AB and FG is congruent to BC, then we also say that AC is the sum of DE and FG.

[1] This description of congruence is intended to be intuitive.



Ray: A ray is a part of a line consisting of all the points on one side of a point on the line, called the endpoint of the ray. We name rays by naming the endpoint, followed by any other point on the ray.

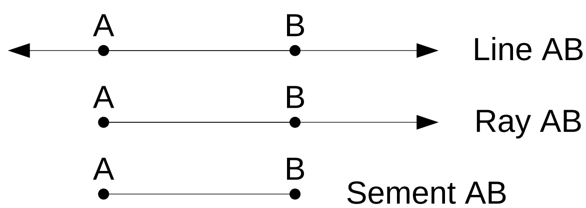
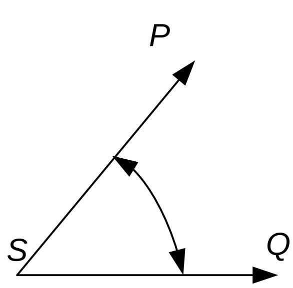


Fig 1.1: A line, a ray and a line segment

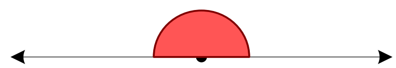
If *A*, *B* and *C* lie on a line, with *B* between *A* and *C*, then ray *BA* and ray *BC* are said to be opposites.

Angles

Angle, vertex and sides: Two rays *SP* and SQ with the same endpoint *S* form an angle. The common endpoint *S* is called the vertex of the angle and the rays are called the sides. Angles are named with three letters, where the middle letter indicates the vertex and the other two are points on the sides. The angle below is called $\angle PSQ$ *or* $\angle QSP$.

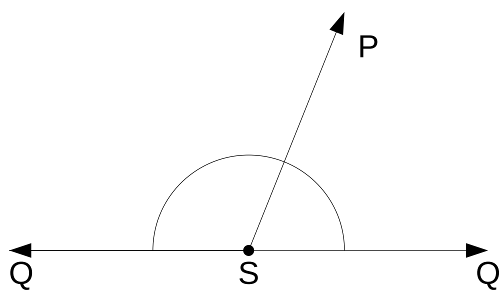


Straight angles: Two rays with a common endpoint that lie in the same line but point in opposite directions are called a straight angle. In other words, a ray and its opposite form a straight angle.



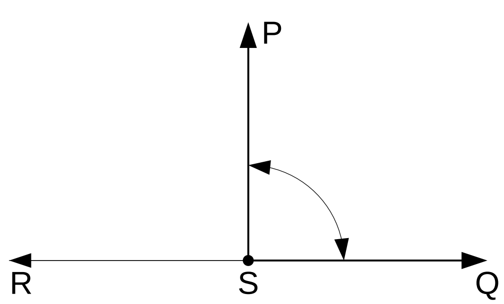
Interior of an angle: If $\angle PSQ$ is not straight, the pointed region bounded by the sides is called the interior.

Supplement of an angle: Suppose $\angle PSQ$ is angle that is not straight. Suppose R is on line SQ and on the opposite side of this line from Q. Then angle $\angle PSR$ is called a supplement of $\angle PSQ$.



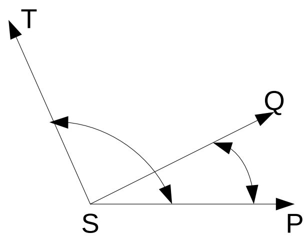
Congruence of angles: Two angles are said to be congruent if they have the same "spread" or rotation. Given two angles, we can tell if they are congruent by placing one on top of the other. If we can do this in such a manner that the vertices and sides match, they are congruent.[2]

Right Angle: An angle that is congruent to its supplement is called a right angle. In the piture below, $\angle PSQ$ is a supplement of $\angle RSP$ and $\angle PSQ$ is congruent to $\angle RSP$. Therefore, $\angle PSQ$ is a right angle.

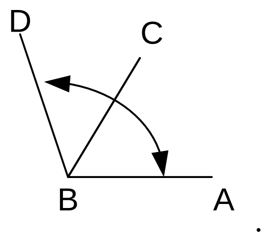


Comparing angles: Given $\angle PSQ$ and $\angle PST$ with ray SP as a common side, we say: $\angle PSQ$ is less than $\angle PST$ if ray SQ is in the interior of $\angle PST$.

[2] This description is again intuitive. A more formal discussion of congruence will be given later.



Adding Angles: Two angles may be added to form a larger angle. In the following picture, ∠ABC and ∠CBD are added to form ∠ABD.



Note: …. We will only work with angles that add to an angle that is less than a straight angle.

<u>Vertical Angles</u>

Define

Show that vertical angles are congruent

Chapter 2: Triangles and congruence

<u>Triangles</u>

Triangle: A triangle is a figure formed by three segments joining three points that are not on a straight line. The points are called the vertices and the segments are called the edges.

A triangle is called *right-angled* if one angle is right. An e*quilateral* triangle is a triangle that has three congruent sides. An *isosceles triangle* is a triangle that has two congruent sides.

<u>Congruence of Triangles</u>

Two triangles are said to be congruent when they have the same size and shape. To determine visually if two triangles are congruent, we lay one on top of the other. If we can do this in such a way that all three vertices match, then the triangles are congruent. Note that sometimes this can be achieved by sliding and rotating only, but sometimes we need to flip a triangle over to get a match.

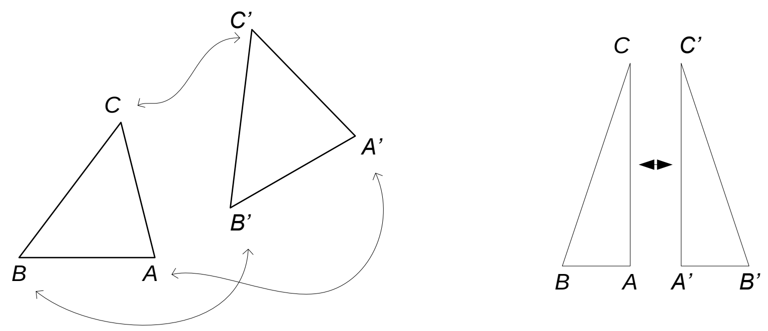


Fig 1.1 Congruent triangles

Notation for triangles. Triangles are often referred to by naming their vertices. If we are only speaking about one triangle, then it does not matter much in what order we list the vertices. On the other hand, if we are making comparisons between two triangles, it may be important to keep track of how the vertices are matched.[3]

For this reason, when we talk about congruence, we choose a notation that respects the order in which the vertices are listed. When we say triangle ABC is congruent to triangle XYZ, we mean that we can lay A on top of X, B on top of Y and C on top of Z and see the two triangles coincide.

To put this another way, we say triangle ABC is congruent to triangle A'B'C' if:

I. AB = A'B'
II. AC =A'C'
III. BC=B'C'
IV. ∠CBA=∠C'B'A'
V. ∠BAC=∠B'A'C'
VI. ∠ACB=∠A'B'C'

Tests for congruence

It is not necessary to check all of the conditions in the list above to determine if two triangles are congruent. If any five of them are true, then so is the remaining one. In fact, there are many collections of just three conditions that imply all the rest.

[3] Given two triangles, each with sides of 3, 4 and 5 inches, there is only one way to match the vertices to show the congruence. Two isosceles triangles that are congruent can be matched in two ways. Two equilateral triangles with the same edge length can be matched in 6 ways.

1. **SAS (Side Angle Side)** Two triangles are congruent if the length of two sides of one is equal to the length of the corresponding two sides of the other and the angles between these sides are same.

2. **SSS (Side, Side, Side)** Two triangles are congruent if the length of three sides of one is equal to the length of the corresponding three sides of the other.

3. **ASA (Angle Side Angle)** Two triangles are congruent if the length of a side of one is equal to the length of the corresponding side of the other and two angles of one are equal to the corresponding angles of the other.

4. Two triangles are congruent if the length of two sides of one is equal to the length of the corresponding two sides of the other and the angles opposite the larger sides are equal.

Properties of triangle

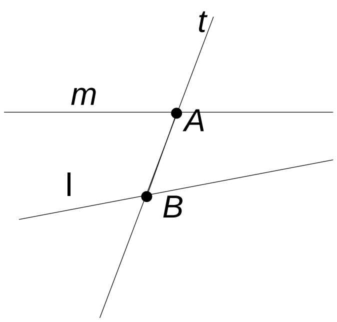
- The sum of the lengths of two sides is always greater than the third side.

- The largest side is opposite to the largest angle and the shortest side is opposite to the smallest angle.

Transversals, parallels and the angles in a triangle

Transversals

Transversal: Let *l and m* be two different lines in a plane. A third line *t* that contains one point (call it *A*) of *l* and one point (call it *B*) of *m* is said to be a transversal of *l* and *m*.



Given any transversal, there are 8 angles of interest, as shown in the following diagram:

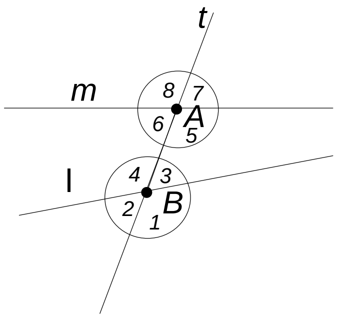


Fig 1.2 The angles formed by a transversal

Interior and exterior angles: In the diagram above, angles 3, 4, 5, and 6 are interior and 1, 2, 7, and 8 are exterior. In general, if *A* and *B* are the points of intersection of a transversal, the angles having ray *AB* as an edge or ray *BA* as edges are called interior. The others are called extrior.

Pairs of angles of a transversal. Two angles of a transversal are said to be alternate if their interiors are on opposite sides of the transversal line *t*. They are said to be corresponding if their interiors are on the same side of *t* and one is interior and one is exterior.

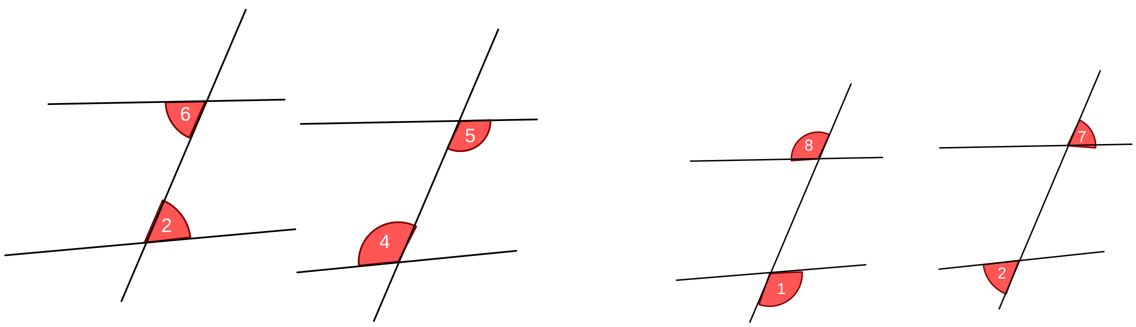


Fig 1.2 Alternate Angles. On the left is a pair of alternate exterior angles, and on the right a pair of alternate interior angles.

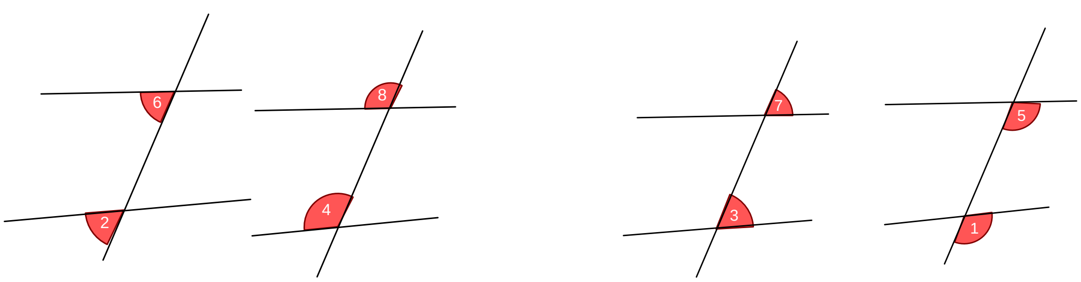


Fig 1.3 Corresponding Angles. Four pairs of corresponding angles

Fact: In any transversal, if one pair of opposite interior angles is congruent, then so are the angles in the other pair, vice versa if one opposite exterior angle pair is congruent then the other opposite exterior angle is also congruent.

Parallel Lines

Parallel lines: Two lines in a plane that do not meet are said to be parallel.

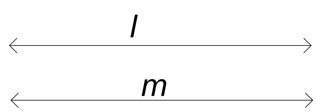
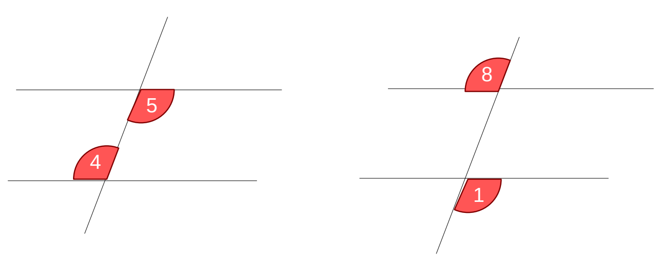


Fig 1.6: parallel lines *PQ* and *RS*

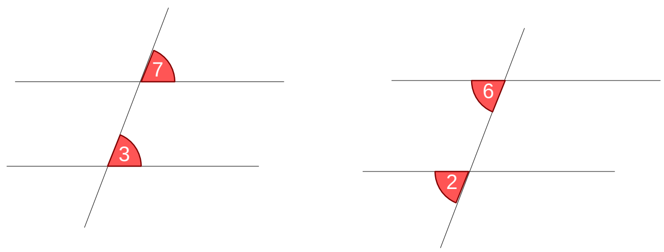
Parallel Postulate: If lines l and m and parallel, then in any transversal, any two opposite interior angles are congruent.

The Parallel Postulate is a basic fact about parallel lines. It can be stated in many different ways using the act

Alternate angles are equal: If alternate angles (4 and 5 and 1 and 8) are equal then lines are parallel and vice versa.



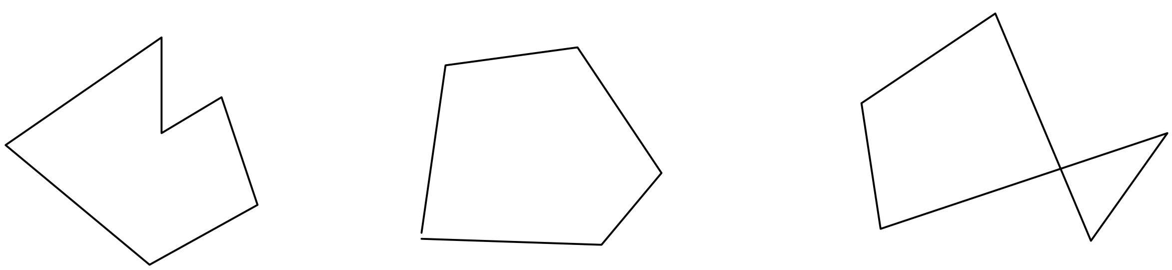
Corresponding angles are equal: If corresponding angles (3 and 7 and 2 and 6) are equal then the lines are parallel and vice versa.



Chapter 4:

Polygons

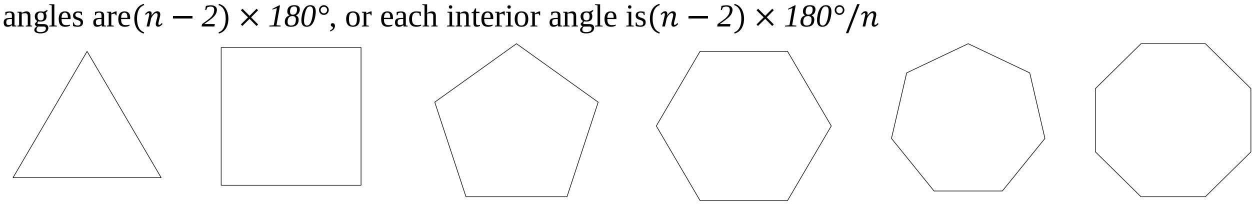
Closed plane figures with straight edges are called polygons. The **vertices** of polygons are the points where the edges meet, the segment connecting neighboring vertices are the **sides** of polygon and the number of sides equal the number of vertices. Triangles are three sided, quadrilaterals are four sided, pentagons are five sided etc. if two non adjacent sides of a polygon intersect, it is called **reflex**. If segments connecting any two points inside the polygon always lye inside that polygon and all angles less than 180° then it is **convex**. Otherwise it is **concave** polygon.



concave convex reflex

Regular Convex Polygons

Regular convex polygons of N sides have N equal angles. In which case the sum of the interior angles are$(n-2)\times 180°$, or each interior angle is$(n-2)\times 180°/n$

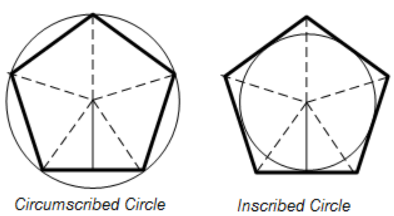


Triangle Square Pentagon Hexagon Heptagon Octagon

Table: The polygons and their interior angles.

Polygon Name	Sides	Interior angle	Exterior Angle
Triangle	3	$(3-2) \times 180°/3 = 60°$	$120°$
Square	4	$(4-2) \times 180°/4 = 90°$	$90°$
Pentagon	5	$(5-2) \times 180°/5 = 108°$	$72°$
Hexagon	6	$(6-2) \times 180°/6 = 120°$	$60°$
Heptagon	7	$(7-2) \times 180°/7 = 128.5°$	$51.50°$
Octagon	8	$(8-2) \times 180°/8 = 135°$	$45°$

. A circle can be inscribed in it in such a way that sides of the polygon are tangent to the circle and another circle can be circumscribed in such a way that sides are cords to the circle.



Quadrilaterals

Quadrilaterals are four sided figures where the sum of the exterior angles is 360°. Square, rectangle, rhomboid, trapezoid, trapezium, kites are shapes of quadrilaterals.

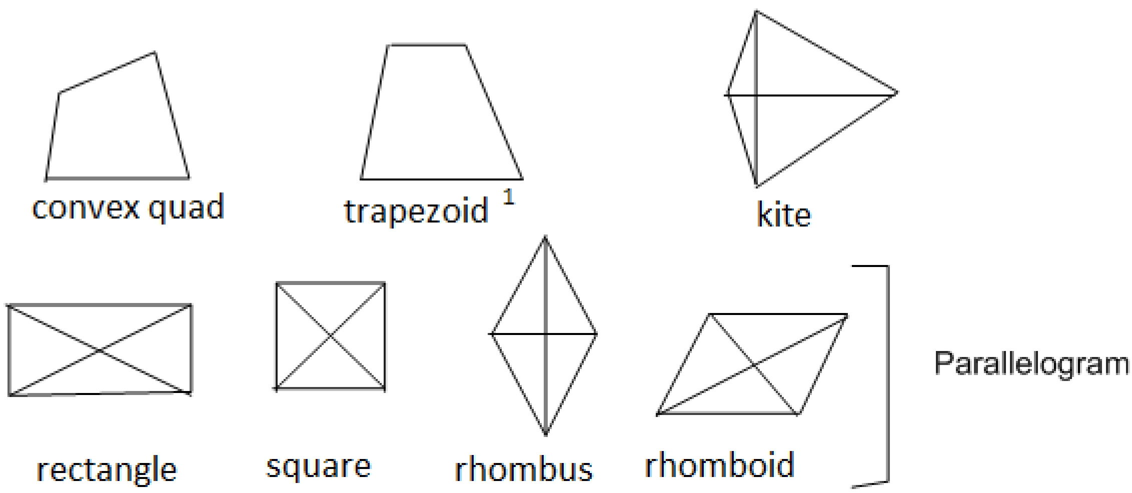
Types of quadrilaterals

Classification by length of sides

General quadrilaterals	All four sides of different length
Kite	Two pairs of equal adjacent sides
Parallelogram	Two pairs of equal opposite sides
Rhombus	All four sides equal

Classification by relative position of sides

General quadrilaterals	No pair of parallel sides
Trapezium	One pair of parallel sides
Parallelogram	Two pair of parallel sides



1. Trapezium (British English) or trapezoid (North America.): one pair of opposite sides is parallel.

Trapezoid: A quadrilateral having *exactly* one pair of parallel sides.

Convex quad: No pair is parallel

Kite: two pairs of equal adjacent sides

Rectangle: All sides form right angles but different size pair

Parallelogram: two pairs of parallel sides

Square: All equal sides making right angles

Rhombus: All four sides equal but no right angle

Rhomboid: two pairs of parallel unequal sides but no right angle

Exercise

How many triangles are formed if four non collinear points connected together with non intersecting straight lines?

How many triangles are formed if five non collinear points connected together with non intersecting straight lines?

How many different angles are formed when two straight lines intersect each other?

What kind of a triangle ABC when sides AB=BC=CD?

What kind of a triangle ABC when sides AB=BC+CD?

What kind of a quadrilateral when two parallel lines are intersected by two non parallel lines?